

Graph $f(x) = \frac{x}{(2-x)^3} - 3$ using the process shown in lecture and in the website handout.

SCORE: ____ / 40 PTS

The first and second derivatives are $f'(x) = (2-x)^{-3} + 3x(2-x)^{-4}$ and $f''(x) = 6(2-x)^{-4} + 12x(2-x)^{-5}$.

Do NOT find x-intercepts.

Complete the table below, after showing relevant work (except for entries marked ★).

You will NOT receive credit for the entries in the table if the relevant work is missing.

★ Domain	★ Discontinuities	y - intercepts <u>ONLY</u>	One sided limits at each discontinuity (write using proper limit notation)	
$x \neq 2$	$x = 2$	$(0, -3)$	$\lim_{x \rightarrow 2^-} f(x) = \infty$	$\lim_{x \rightarrow 2^+} f(x) = -\infty$
Horizontal Asymptotes	Intervals of Increase	Intervals of Decrease	Intervals of Upward Concavity	Intervals of Downward Concavity
$y = -3$	$(-1, 2), (2, \infty)$	$(-\infty, -1)$	$(-2, 2)$	$(-\infty, -2), (2, \infty)$
Vertical Tangent Lines	Horizontal Tangent Lines	Local Maxima	Local Minima	Inflection Points
NONE	$x = -1$	NONE	$(-1, -3\frac{1}{27})$	$(-2, -3\frac{1}{32})$

$$f(0) = \frac{0}{2^3} - 3 = -3$$

$$\lim_{x \rightarrow 2^+} \left(\frac{x}{(2-x)^3} - 3 \right) = -\infty \quad \left(\frac{2}{0^+} - 3 \rightarrow -\infty - 3 \rightarrow -\infty \right)$$

$$\lim_{x \rightarrow 2^-} \left(\frac{x}{(2-x)^3} - 3 \right) = \infty \quad \left(\frac{2}{0^-} - 3 \rightarrow \infty - 3 \rightarrow \infty \right)$$

$$\lim_{x \rightarrow \pm\infty} \left(\frac{x}{(2-x)^3} - 3 \right) = \left(\lim_{x \rightarrow \pm\infty} \frac{1}{-3(2-x)^2} \right) - 3$$

$$\frac{\infty}{\pm\infty} - 3 = 0 - 3 = -3$$

$$f'(x) = (2-x)^{-4} (2-x+3x)$$

$$= (2-x)^{-4} (2x+2)$$

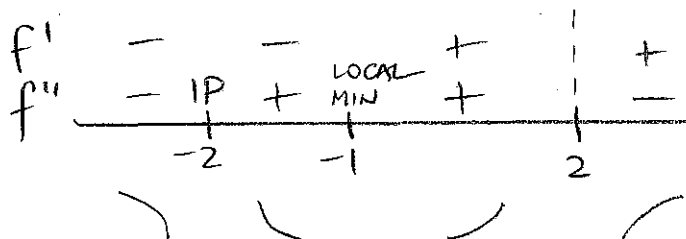
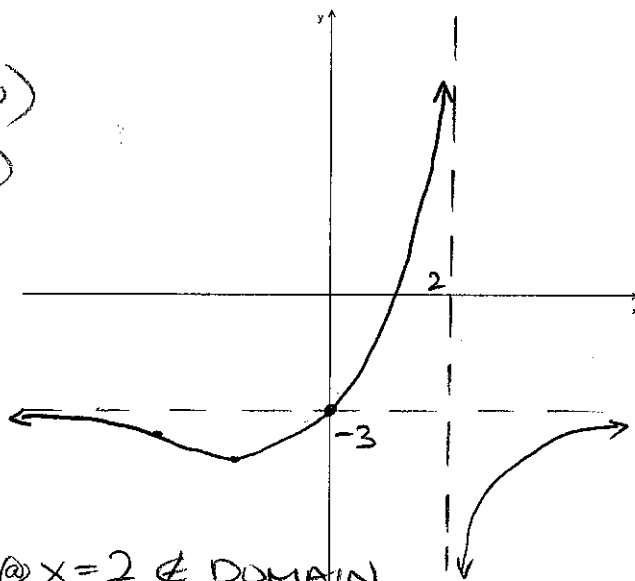
$$= 2(2-x)^{-4} (x+1) \text{ UNDEFINED @ } x=2 \notin \text{DOMAIN}$$

$$= 0 \text{ @ } x=-1$$

$$f''(x) = 6(2-x)^{-5} (2-x+2x)$$

$$= 6(2-x)^{-5} (2+x) \text{ UNDEFINED @ } x=2 \notin \text{DOMAIN}$$

$$= 0 \text{ @ } x=-2$$



$f(x)$ is a continuous function whose derivative $f'(x)$ is shown on the right.

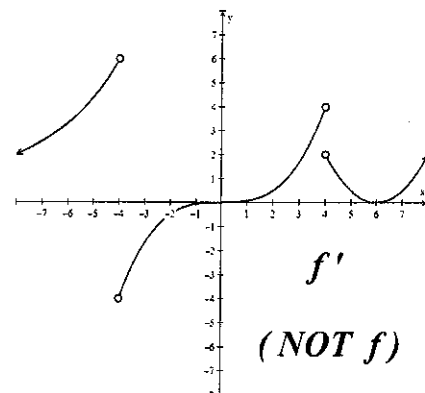
SCORE: ____ / 24 PTS

The following questions are about the function f , **NOT THE FUNCTION f'** .

- [a] Find the x -coordinates of all inflection points of f .

Justify your answer very briefly.

f' CHANGES FROM INCR TO DECR: $x=4$
 DECR INCR: $x=6$



- [b] Find the intervals over which f is decreasing.

Justify your answer very briefly.

$f' < 0 : (-4, 0)$

- [c] Find all critical numbers of f , and state what the First Derivative Test tells you about each one.

Justify your answer very briefly.

$f' = 0 : 0$	f' CHANGES FROM - TO + $\rightarrow$ LOCAL MIN
6	+ + $\rightarrow$ NOT EXTREMA
f' DNE: -4	+ - $\rightarrow$ LOCAL MAX
4	+ + $\rightarrow$ NOT EXTREMA

$f(x)$ is a continuous and differentiable function whose second derivative $f''(x)$ is shown on the right.

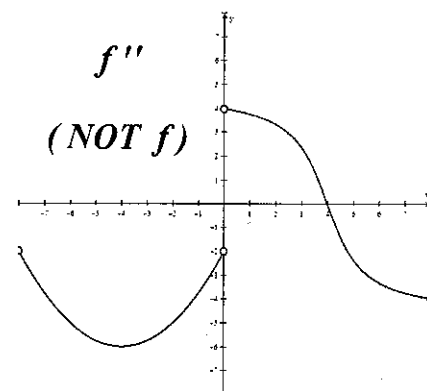
SCORE: ____ / 12 PTS

The following questions are about the function f , **NOT THE FUNCTION f''** .

- [a] If $f'(6) = 0$,
 what does the Second Derivative Test tell you about the point $(6, f(6))$?

Justify your answer very briefly.

$f''(6) < 0 \rightarrow$ LOCAL MAX



- [b] Find the intervals over which f is concave up.

Justify your answer very briefly.

$f'' > 0 : (0, 4)$

Find $\int \frac{(3x^3 - 2)^2}{5x^4} dx$.

SCORE: ____ / 15 PTS

$$= \int \frac{9x^6 - 12x^3 + 4}{5x^4} dx$$

$$= \int \left(\frac{9}{5}x^2 - \frac{12}{5}x^{-1} + \frac{4}{5}x^{-4} \right) dx$$

$$= \frac{9}{5} \frac{1}{3}x^3 - \frac{12}{5} \ln|x| + \frac{4}{5} \left(-\frac{1}{3}\right)x^{-3} + C$$

$$= \frac{3}{5}x^3 - \frac{12}{5} \ln|x| - \frac{4}{15}x^{-3} + C$$

Find $\lim_{x \rightarrow 0^+} \sqrt{x} \ln x$. $0, -\infty$

SCORE: ____ / 15 PTS

$$= \lim_{x \rightarrow 0^+} \frac{\ln x}{x^{-\frac{1}{2}}} \quad \frac{-\infty}{\infty}$$

$$= \lim_{x \rightarrow 0^+} \frac{\frac{1}{x}}{-\frac{1}{2}x^{-\frac{3}{2}}}$$

$$= \lim_{x \rightarrow 0^+} -2x^{\frac{1}{2}} = 0$$

Does Rolle's Theorem apply to the function $f(x) = \sqrt[3]{x^2 - 4x + 3}$ on the interval $[-1, 5]$?

SCORE: ____ / 15 PTS

(That is, are all conditions of Rolle's Theorem true for $f(x) = \sqrt[3]{x^2 - 4x + 3}$ on the interval $[-1, 5]$?)
If yes, find the value of c guaranteed by Rolle's Theorem. If no, explain why not.

$$f'(x) = \frac{1}{3} (x^2 - 4x + 3)^{-\frac{2}{3}} (2x - 4)$$

$$\text{IS UNDEFINED @ } x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$x = 1, 3 \in [-1, 5]$$

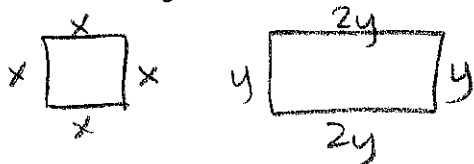
f IS NOT DIFFERENTIABLE ON $[-1, 5]$

SO ROLLE'S THEOREM DOES NOT APPLY

A piece of wire 34 inches long is cut into two pieces. One piece is bent into a square and the other piece is bent into a rectangle whose width is twice its length. Each side of the square and of the rectangle must be at least 1 inch long. SCORE: ____ / 30 PTS

You must use calculus to solve the following problems.

[a] What is the largest total area that can be enclosed in the two shapes?



MAXIMIZE $A = x^2 + 2y^2$

$$4x + 6y = 34$$

$$2x + 3y = 17$$

$$x \geq 1 \Rightarrow y \leq 5$$

$$x = \frac{1}{2}(17 - 3y)$$

$$A = \frac{1}{4}(17 - 3y)^2 + 2y^2 \text{ ON } y \in [1, 5]$$

$$A' = \frac{1}{2}(17 - 3y)(-3) + 4y \text{ IS DEFINED ON } [1, 5]$$

$$= \frac{9}{2}y - \frac{51}{2} + 4y = 0$$

$$9y - 51 + 8y = 0$$

$$17y = 51$$

$$y = 3 \rightarrow x = 4$$

y A

1 $\frac{1}{4}(14^2) + 2(1^2) = 49 + 2 = 51$

3 $\frac{1}{4}(8^2) + 2(3^2) = 16 + 18 = 34$

5 $\frac{1}{4}(2^2) + 2(5^2) = 1 + 50 = 51$

THE LARGEST AREA IS 51 SQUARE INCHES

[b] Find the dimensions of the square and the rectangle which give the largest total area.

EITHER 7" x 7" AND 1" x 2"

OR 1" x 1" AND 5" x 10"